An Exhaustive Classification of Photonic Topological Insulators

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in collarboration with Giuseppe De Nittis

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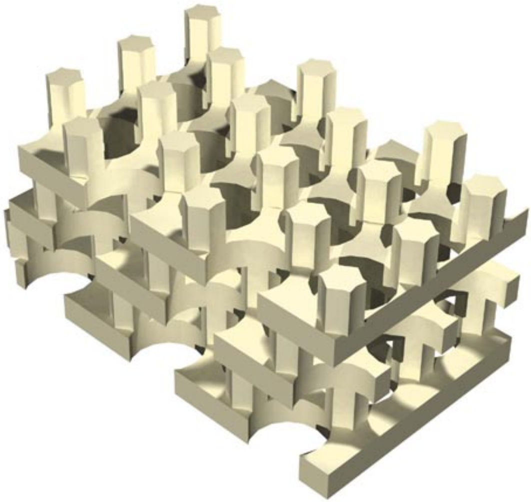
Talk based on

Collaboration with **Giuseppe De Nittis**

- *On the Role of Symmetries in the Theory of Photonic Crystals*
  to appear in the Annals of Physics
- *The Perturbed Maxwell Operator as Pseudodifferential Operator*
  Documenta Mathematica **19**, 2014
- *Effective Light Dynamics in Perturbed Photonic Crystals*
  to appear in Comm. Math. Phys.



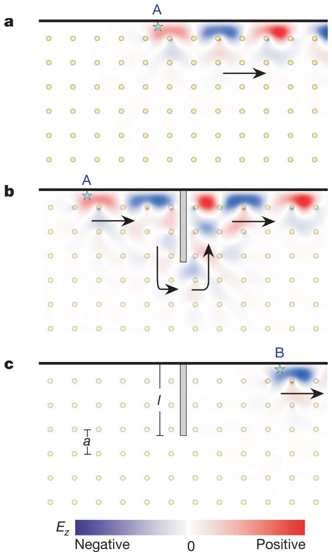
Johnson & Joannopoulos (2004)

Quantum-light analogies

1987-2005 Research focuses on photonic crystals with *photonic band gap*

2005-now Two seminal work by Raghu & Haldane: study of *topological* properties

Quantum-light analogies



Wang et al (2009)

Quantum-light analogies

»A photonic crystal is to light what a crystalline solid is to an electron.«

Quantum-light analogies

- Photonic bulk-edge correspondences

$\downarrow$

- Identify topological observables $O = T +$ error

$\downarrow$

- Find all topological invariants T

$\downarrow$

- Classification of PhCs by symmetries

Quantum-light analogies

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- Identify topological observables $O = T + \text{error}$

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- Classification of PhCs by symmetries

Quantum-light analogies

- Photonic bulk-edge correspondences

$\downarrow$

- Identify topological observables $O = T +$ error

$\downarrow$

- Find all topological invariants T

$\downarrow$

- **Classification of PhCs by symmetries** $\rightsquigarrow$ **today**

The Material Weights

Properties of the material enter through phenomenological

$$W(x)^{-1} = \begin{pmatrix} \varepsilon(x) & \chi(x) \\ \chi(x)^* & \mu(x) \end{pmatrix} \in \mathrm{Mat}_{\mathbb{C}}(6)$$

Assumption (Material weights)

1. $0 < c\,\mathbf{1} \leq W^{-1} \leq C\,\mathbf{1}$ *(W exists, satisfies same condition)*
2. $W^* = W$ *(lossless)*
3. *W frequency-independent (medium linear)*

Maxwell equations as Schrödinger equation

1. *Field energy*

$$\mathcal{E}(\mathbf{E},\mathbf{H}) = \frac{1}{2}\int_{\mathbb{R}^3} \mathrm{d}x \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix} \cdot \begin{pmatrix} \varepsilon(x) & \chi(x) \\ \chi(x)^* & \mu(x) \end{pmatrix} \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix}$$

2. *Dynamical equations*

$$\begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla_x \times \mathbf{H} \\ +\nabla_x \times \mathbf{E} \end{pmatrix}$$

3. *No sources*

$$\begin{pmatrix} \mathrm{div} & 0 \\ 0 & \mathrm{div} \end{pmatrix} \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = 0$$

Maxwell equations as Schrödinger equation

1. *Field energy*

$$\mathcal{E}\big(\mathbf{E},\mathbf{H}\big) = \mathcal{E}\big(\mathbf{E}(t),\mathbf{H}(t)\big)$$

2. *Dynamical equations*

$$\begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla_x \times \mathbf{H} \\ +\nabla_x \times \mathbf{E} \end{pmatrix}$$

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2. *Dynamical equations*

$$\begin{pmatrix} \varepsilon\,\partial_t \mathbf{E} + \chi\,\partial_t \mathbf{H} \\ \chi^*\,\partial_t \mathbf{E} + \mu\,\partial_t \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla_x \times \mathbf{H} \\ +\nabla_x \times \mathbf{E} \end{pmatrix}$$

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3. *No sources*

$$\begin{pmatrix}\nabla \cdot \big(\varepsilon\,\mathbf{E} + \chi\,\mathbf{H}\big)\\ \nabla \cdot \big(\chi^*\,\mathbf{E} + \mu\,\mathbf{H}\big)\end{pmatrix} = 0$$

Maxwell equations as Schrödinger equation

1. *Field energy* $(\mathbf{E}, \mathbf{H}) \in L^2_w(\mathbb{R}^3, \mathbb{C}^6)$ with energy norm

$$\big\|(\mathbf{E}, \mathbf{H})\big\|^2_{L^2_w} := \int_{\mathbb{R}^3} \mathrm{d}x \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix} \cdot \begin{pmatrix} \varepsilon(x) & \chi(x) \\ \chi(x)^* & \mu(x) \end{pmatrix} \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix}$$

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$$\big\| (\mathbf{E}, \mathbf{H}) \big\|^2_{L^2_w} = 2\,\mathcal{E}(\mathbf{E}, \mathbf{H})$$

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$$\mathrm{i} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} \begin{pmatrix} 0 & +\mathrm{i}\nabla^\times \\ -\mathrm{i}\nabla^\times & 0 \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

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$$\mathrm{i}\frac{\partial}{\partial t}\underbrace{\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}}_{=\Psi} = \underbrace{\begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix}^{-1} \begin{pmatrix} 0 & +\mathrm{i}\nabla^\times \\ -\mathrm{i}\nabla^\times & 0 \end{pmatrix}}_{=M_w} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$$

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$$\mathrm{i} \frac{\partial}{\partial t} \Psi = M_w \Psi$$

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The Maxwell operator

$$M_w = \begin{pmatrix} \varepsilon(x) & \chi(x) \\ \chi(x)^* & \mu(x) \end{pmatrix}^{-1} \begin{pmatrix} 0 & +\mathrm{i}\nabla^\times \\ -\mathrm{i}\nabla^\times & 0 \end{pmatrix}$$
$$= W\,\mathrm{Rot} = W\left(-\sigma_2 \otimes \nabla^\times\right)$$

M_w selfadjoint on *weighted* $L^2_w(\mathbb{R}^3, \mathbb{C}^6)$
$\Rightarrow e^{-\mathrm{i}tM_w}$ unitary, yields conservation of energy

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Symmetries of the free Maxwell operator Rot

$$\mathrm{Rot} = \begin{pmatrix} 0 & +\mathrm{i}\nabla^{\times} \\ -\mathrm{i}\nabla^{\times} & 0 \end{pmatrix} = -\sigma_2 \otimes \nabla^{\times}$$

Symmetries

For $n = 1, 2, 3$

1. $T_n = \sigma_n \otimes \mathrm{id}$ (linear)
2. Complex conjugation C (antilinear)
3. $J_n = T_n C$ (antilinear)

Symmetries of the free Maxwell operator Rot

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Symmetries of the free Maxwell operator Rot

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Action of symmetries on Rot

1. $C\,\mathrm{Rot}\,C = -\mathrm{Rot}$
2. $T_n\,\mathrm{Rot}\,T_n = -\mathrm{Rot}, n = 1, 3$
   $T_2\,\mathrm{Rot}\,T_2 = +\mathrm{Rot}$
3. $J_n\,\mathrm{Rot}\,J_n = +\mathrm{Rot}, n = 1, 3$
   $J_2\,\mathrm{Rot}\,J_2 = -\mathrm{Rot}$

Classification of Maxwell operator in matter

Product structure of $M_w = W \, \mathrm{Rot}$:

$$U \, W \, U^* = \pm W \implies U \, M_w \, U^* = \pm M_w$$

(Signs may be different)

Symmetries $U = T_n, C, J_n, n = 1, 2, 3$

Classification of Maxwell operator in matter

Product structure of $M_w = W \, \mathrm{Rot}$:

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(Signs may be different)

Symmetries $U = T_n, C, J_n, n = 1, 2, 3$

Classification of Maxwell operator in matter

$$U\,W\,U^* = \pm W \iff U\,W^{-1}\,U^* = \pm W^{-1}$$

$$U\,M_W\,U^* = \pm M_W \implies \text{conditions on } W^{-1} = \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix}$$

Classification of Maxwell operator in matter

$$U\,W\,U^* = \pm W \quad \Longleftrightarrow \quad U\,W^{-1}\,U^* = \pm W^{-1}$$

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Relevance of symmetries for classification

Mathematically irrelevant symmetries, e. g.

1. $T_n \, M_w \, T_n = +M_w$ (linear, commuting)
2. Parity $(P\Psi)(x) := \Psi(-x)$ (linear, anticommuting)

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Symmetry leads to unphysical conditions on weights, e. g.

$$C\,W\,C = -W \quad \Leftrightarrow \quad C\,M_w\,C = +M_w$$

implies $\varepsilon \in \mathrm{i}\mathbb{R}$, $\mu \in \mathrm{i}\mathbb{R}$, $\chi \in \mathrm{i}\mathbb{R}$ (keep in mind $\varepsilon = \varepsilon^*$ and $\mu = \mu^*$)

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CAZ \ realized			
A	*none*		
AIII	$T_1 \equiv \chi$	$T_2 \equiv \chi$	$T_3 \equiv \chi$
AI	$J_1 \equiv +\mathrm{TR}$	$J_3 \equiv +\mathrm{TR}$	$C \equiv +\mathrm{TR}$
AII	$J_2 \equiv -\mathrm{TR}$		
D	$J_1 \equiv +\mathrm{PH}$	$J_3 \equiv +\mathrm{PH}$	$C \equiv +\mathrm{PH}$
C	$J_2 \equiv -\mathrm{PH}$		

realized / **CAZ**			
BDI	$J_1 \equiv +$TR $C \equiv +$PH	$C \equiv +$TR $J_1 \equiv +$PH	$J_3 \equiv +$TR $C \equiv +$PH
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Symmetries present	CAZ class	Reduced K-group in dimension			
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Cartan-Altland-Zirnbauer classification of Maxwell operators

- 9 out of 10 CAZ classes theoretically realizable
- 24 different realizations of CAZ classes
- Not all realizations physically relevant
- *At least 5 out of those 9 CAZ classes considered in physics*
- Electromagnetic fields are not fermions
- Reduced *K*-groups: *anticipation of topological invariants*

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Material weights of ordinary materials

$$W^{-1} = \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} = \begin{pmatrix} \mathrm{Re}\,\varepsilon & 0 \\ 0 & \mathrm{Re}\,\mu \end{pmatrix}$$

Material weights of ordinary (non-gyrotropic) materials

- $\varepsilon, \mu \in \mathbb{R}$
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- Relies on $\chi = 0$
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Incorrect classification of C as time-reversal symmetry

Classification in second-order framework

Physicists usually start with $\left(\partial_t^2 + M_w^2\right)\Psi(t) = 0$
$\rightsquigarrow$ action of symmetry

$$C\, M_w^2\, C = (-1)^2\, M_w^2 = M_w^2$$

"$\Rightarrow$" C is a *time-reversal symmetry*
"$\Rightarrow$" Ordinary (non-gyrotropic) materials are of **class AI**
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Conclusions are **false**!

Incorrect classification of C as time-reversal symmetry

Example (Free Dirac equation)

- Write Dirac equation as $\mathrm{i}\,\partial_t \Psi = H_D \Psi$ where

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TR and PH symmetry *both* incorrectly appear as "TR symmetry"

2nd-order framework distinguishing CAZ types of symmetries (TR vs. PH) impossible!

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Proper identification of CAZ classes

ordinary (non-gyrotropic)	**"topologically non-trivial" (gyrotropic)**
$\varepsilon \neq \mu$, $\chi = 0$	
$\varepsilon, \mu \in \mathbb{R}$ $T \equiv \chi$, $C \equiv +\mathrm{PH}$	$\varepsilon \in \mathbb{C} \vee \mu \in \mathbb{C}$ $T \equiv \chi$
class BDI (not class AI)	**class AIII** (not class A)

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Absence of topological effects in class BDI?

Topological effects in BDI?
Seemingly contradicts experimental observations.

Absence of topological effects in class BDI?

Assumption

W periodic

Frequency band picture

- $M_W \cong \int_{\mathbb{B}}^{\oplus} \mathrm{d}k \, M_W(k)$
- gives rise to Bloch functions and frequency bands

$$M_W(k) \varphi_n(k) = \omega_n(k) \, \varphi_n(k)$$

- $C \, M_W(k) \, C = -M_W(-k)$
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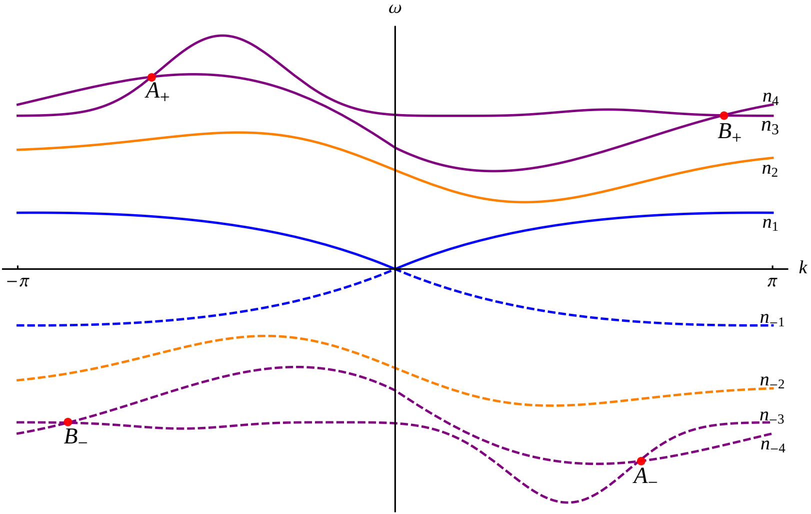

Only *C*-symmetry present, *T*-symmetry broken!

Absence of topological effects in class BDI?

Theorem (De Nittis-L. (2013))

ω_n isolated band

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$\Rightarrow$ Absence of topological effects **due to Chern classes** for **real** electromagnetic fields

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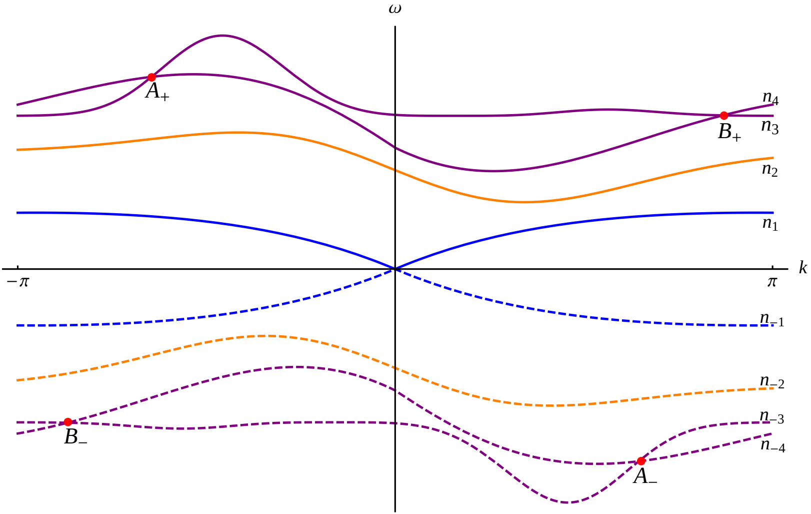

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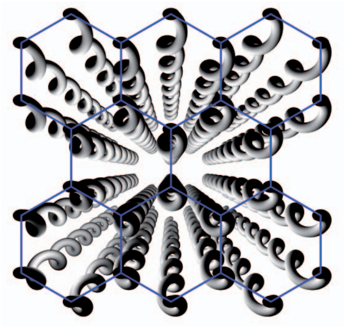
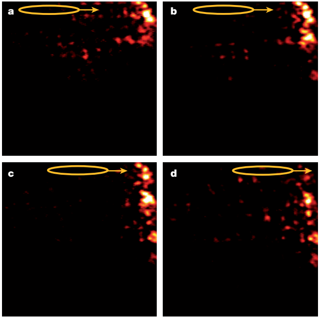
Periodic waveguide arrays





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- *Backscattering-free unidirectional boundary currents* measured
- Ordinary material (silica) $\Rightarrow$ **class BDI**
- Experiments explained by use of effective models

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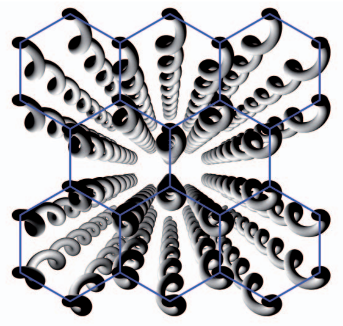

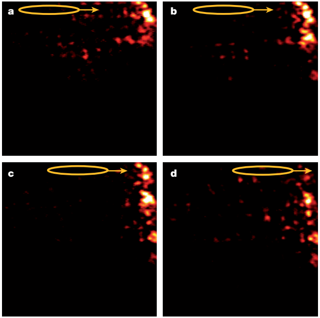

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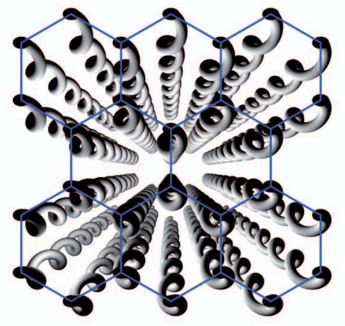

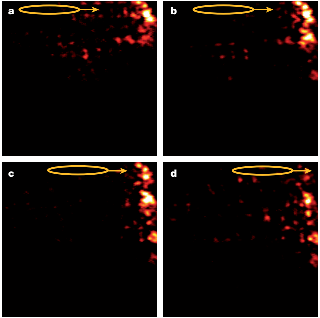

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$\downarrow$

- Identify bulk & edge topological observables $O = T + \text{error}$

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- Find all topological invariants T for PhCs of given CAZ class

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- Classification of PhCs by symmetries (finished)

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- Establish **bulk-edge correspondences**
- Interplay symmetries and dynamics
  (e. g. twin-band ray optics, effective tight-binding models)
- Periodic waveguide arrays

Thank you for your attention!

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