

Graded Homework Assignment

This homework assignment will be graded and the marks you get will count towards the grade you will receive for this whole lecture series.

Homework Problems

1. Comparison of the Frameworks of Quantum Mechanics and Classical Electromagnetism (12 points)

Give a table that compares the three fundamental notions — states, observables and dynamical equations — of quantum mechanics and classical electromagnetism.

2. Conservation of total field energy in electromagnetism (20 points)

Suppose the material weights

$$W(x) = \begin{pmatrix} \varepsilon(x) & 0 \\ 0 & \mu(x) \end{pmatrix}$$

that describe the properties of the medium satisfy Assumption 3.1.1 from the lecture notes. These then enter Maxwell's equations, the left-hand side of equation (4.0.1) from the lecture notes.

Consider the field energy observable

$$\mathcal{E}(\mathbf{E}, \mathbf{H}) = \frac{1}{2} \int_{\mathbb{R}^3} dx \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix} \cdot W(x) \begin{pmatrix} \mathbf{E}(x) \\ \mathbf{H}(x) \end{pmatrix}.$$

- (1) Let $(\mathbf{E}(t), \mathbf{H}(t)) \in L^2(\mathbb{R}^3, \mathbb{R}^6)$ a solution to Maxwell's equations. Then show that total field energy \mathcal{E} is a conserved quantity by deriving $\mathcal{E}(\mathbf{E}(t), \mathbf{H}(t))$ with respect to time.
- (2) Suppose that we drop Assumption 3.1.1 (a), namely that $W(x)$ takes values in the hermitian matrices. Is total field energy still necessarily conserved? Explain why or why not.
- (3) Suppose we replace Assumption 3.1.1 (c) with there exist constants $0 < C_{\min} < C_{\max} < \infty$ such that

$$C_{\min} \leq |w_1(x)|, \dots, |w_6(x)| \leq C_{\max}.$$

Is \mathcal{E} still a conserved quantity, i. e. does $\mathcal{E}(\mathbf{E}(t), \mathbf{H}(t)) = \mathcal{E}(\mathbf{E}(0), \mathbf{H}(0))$ still hold for all solutions to Maxwell's equations? Explain why or why not.

- (4) Find another way to prove conservation of total field energy.

REMARK: You may assume that you can always use partial integration.

Hand in home work on: Drop box, 27 December 2019