



Tempered Distributions & Green's Functions

Homework Problems

45. The Sokhotski-Plemelj formula

Consider the linear maps

$$\frac{1}{x \pm i0}(\varphi) := \lim_{\varepsilon \searrow 0} \int_{\mathbb{R}} dx \frac{\varphi(x)}{x \pm i\varepsilon}$$
$$(\mathcal{P}\frac{1}{x})(\varphi) := \lim_{\varepsilon \searrow 0} \int_{|x| \geq \varepsilon} dx \frac{\varphi(x)}{x}$$

on $\mathcal{S}(\mathbb{R})$ from problem 44. Show that they define tempered distributions.

Hint: Inspect the solution to problem 44 and work smart, not hard.

46. Green's function for the Poisson equation in $d = 2$

Modify the proof of Theorem 8.3.1 to the case $d = 2$ in order to prove that the Green's function for $-\Delta_x$ in two dimensions is $G(x, y) = -\frac{1}{2\pi} \ln |x - y|$.

Hand in home work on: Thursday, 6 February 2014, before class