Topological Classification of Electromagnetic Media

in collaboration with Giuseppe De Nittis

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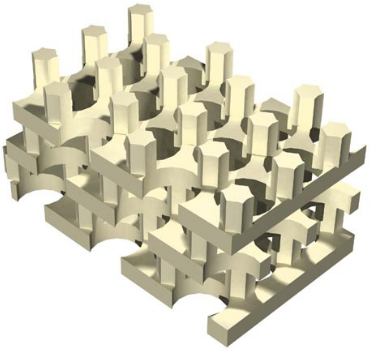
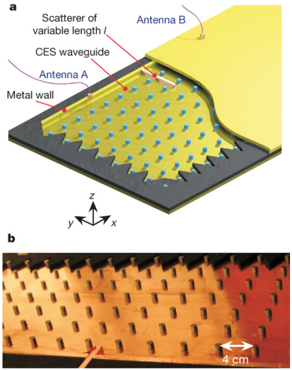
Periodic Light Conductors aka Photonic Crystals



Johnson & Joannopoulos (2004)

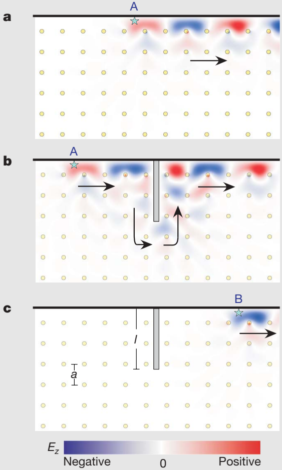


Joannopoulos, Soljačić et al (2009)

Quantum Hall Effect for Light

Predicted theoretically by Raghu & **Haldane** (2005) ...

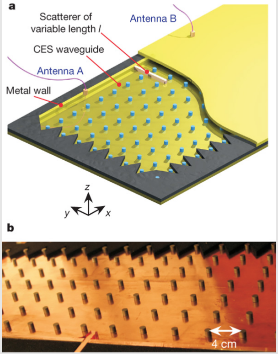
$$\left.\begin{matrix}\begin{pmatrix}\overline{\varepsilon} & 0\\ 0 & \overline{\mu}\end{pmatrix} \neq \begin{pmatrix}\varepsilon & 0\\ 0 & \mu\end{pmatrix}\\ \text{symmetry breaking}\end{matrix}\right\} \Longrightarrow$$



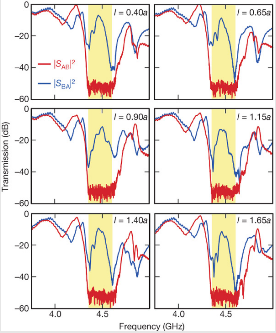
Joannopoulos, Soljačić et al (2009)

Quantum Hall Effect for Light

... and realized experimentally by Joannopoulos et al (2009)

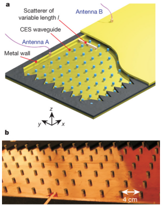


Joannopoulos, Soljačić et al (2009)

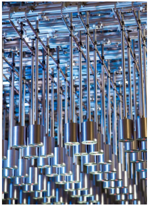


Joannopoulos, Soljačić et al (2009)

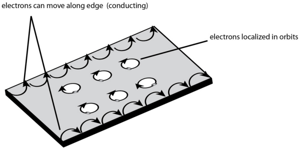
Topological Effects: Phenomenological Similarities



Light



Coupled Oscillators



Quantum

- Periodic structure ⇝ **bulk band gap**
- **Breaking** of time-reversal **symmetries**
- Unidirectional edge modes
- Robust under perturbations

Two Main Questions for Today

1. Is the **Quantum Hall Effect for Light really analogous** to the Quantum Hall Effect?
2. Are there **other topological effects**?

⇝ **Topological classification of electromagnetic media**

Strategy to Obtain Classification

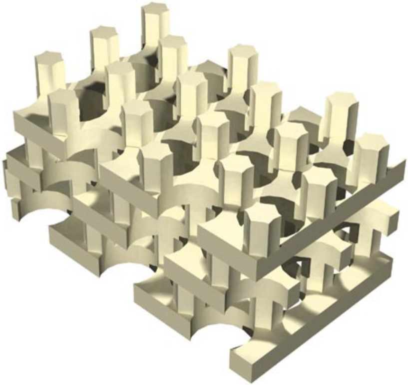
1. Rewrite Maxwell's equations in the form of a Schrödinger equation

   (De Nittis & L., *The Schrödinger Formalism of Electromagnetism and Other Classical Waves* (2017))

2. Apply Cartan-Altland-Zirnbauer classification scheme of (quantum) topological insulators

   (De Nittis & L., *Symmetry Classification of Topological Photonic Crystals* (2017))

Setting: Electromagnetic Waves in Linear Medium



Johnson & Joannopoulos (2004)

Assumption (Material weights)

$$W(x) = \begin{pmatrix} \varepsilon(x) & \chi(x) \\ \chi(x)^* & \mu(x) \end{pmatrix}$$

1. dispersion-free
2. lossless
   ($W(x)^* = W(x)$ hermitian)
3. not a negative or 0 index material
   (the eigenvalues of $W(x)$ are positive and do not reach 0)
4. periodic
   ($W(x+\gamma) = W(x)$ for all lattice vectors γ)

Schrödinger Formalism of Maxwell's Equations

$$\left.\begin{array}{c}\textbf{Real}\text{ transversal states} \\ (\mathbf{E},\mathbf{H}) = 2\mathrm{Re}\,\Psi_+ \\ \begin{pmatrix}\varepsilon & 0 \\ 0 & \mu\end{pmatrix} \frac{\partial}{\partial t}\begin{pmatrix}\mathbf{E} \\ \mathbf{H}\end{pmatrix} = \begin{pmatrix}+\nabla \times \mathbf{H} \\ -\nabla \times \mathbf{E}\end{pmatrix}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{c}\textbf{Complex}\text{ states with } \omega > 0 \\ \Psi = P_+(\mathbf{E},\mathbf{H}) \\ M = W^{-1}\,\mathrm{Rot}\,\big|_{\omega>0} = M^{*W} \\ \mathrm{i}\,\partial_t \Psi = M\Psi\end{array}\right.$$

$$\mathcal{H} = \Big\{ \Psi \in L^2(\mathbb{R}^3,\mathbb{C}^6) \;\big|\; \Psi \text{ is } \omega > 0 \text{ state} \Big\}$$

$$\langle \Phi, \Psi \rangle_W = \int_{\mathbb{R}^3} \mathrm{d}x\, \Phi(x) \cdot W(x)\Psi(x)$$

Energy scalar product

(De Nittis & L., *The Schrödinger Formalism of Electromagnetism and Other Classical Waves* (2017))

Classification of Topological Insulators

1. **Topological class** $\longleftrightarrow$ Discrete symmetries of M
2. $\left.\begin{array}{c}\textbf{Phases} \text{ inside each} \\ \text{topological class}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{c}\text{Labeled by} \\ \text{topological invariants}\end{array}\right.$
3. **Bulk-edge correspondences**

$$\left.\begin{array}{c}\text{physical} \\ \text{observable}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{c}\text{topological} \\ \text{invariant}\end{array}\right.$$

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No Additional Symmetries Assumption

Assumption

Apart from those below the system (i. e. the Maxwell operator M) has **no additional unitary, commuting symmetries**.

Otherwise

1. Block-decompose according to unitary, commuting symmetry.
2. Repeat until no extraneous symmetries are left.
3. Analyze each block separately with the tools used here.

Symmetries Used in Classification

Example

$$T_3 = (\sigma_3 \otimes \mathbb{1})C : \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \mapsto \begin{pmatrix} +\mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} C \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} +\overline{\mathbf{E}} \\ -\overline{\mathbf{H}} \end{pmatrix}$$

Pauli matrix $\sigma_3 = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ in electro-magnetic splitting

Symmetries Used in Classification

Unitary symmetries

$$U_n = \sigma_n \otimes \mathbb{1}, \quad n = 1, 2, 3$$

Antiunitary symmetries

$$T_n = (\sigma_n \otimes \mathbb{1})\, C, \quad n = 0, 1, 2, 3$$

- C is complex conjugation
- $\sigma_0 = \mathbb{1}$ the identity
- σ_1, σ_2 and σ_3 are the Pauli matrices in the **E**-**H** splitting
- U_n and T_n (anti)commute with free Maxwell operator

$$\mathrm{Rot} = \begin{pmatrix} 0 & +\mathrm{i}\nabla^{\times} \\ -\mathrm{i}\nabla^{\times} & 0 \end{pmatrix} = -\sigma_2 \otimes \nabla^{\times}$$

Admissible Symmetries

$$W = \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} = \sum_{n=0}^{3} \sigma_n \otimes w_n$$

where e. g. $w_0 = \frac{1}{2}(\varepsilon + \mu)$ and $w_3 = \frac{1}{2}(\varepsilon - \mu)$

Admissible Symmetries

$$W = \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} = \sum_{n=0}^{3} \sigma_n \otimes w_n$$

Symmetry $V =$	$w_0 =$	$w_1 =$	$w_2 =$	$w_3 =$	*Symmetry Type*
$T_1 = (\sigma_1 \otimes \mathbb{1})C$	Re w_0	Re w_1	Re w_2	i Im w_3	+TR
$U_2 = \sigma_2 \otimes \mathbb{1}$	w_0	0	w_2	0	ordinary
$T_3 = (\sigma_3 \otimes \mathbb{1})C$	Re w_0	i Im w_1	Re w_2	Re w_3	+TR

Admissibility Conditions

- Reality of $(\mathbf{E}, \mathbf{H}) \iff \omega > 0$ fields $\mapsto \omega > 0$ fields $\Longrightarrow V\,M = M\,V$
- Compatibility with energy scalar product $\Longrightarrow V\,W = W\,V$

$\Longrightarrow$ exclude *anti*commuting symmetries

Admissible Symmetries

$$W = \begin{pmatrix} \varepsilon & \chi \\ \chi^* & \mu \end{pmatrix} = \sum_{n=0}^{3} \sigma_n \otimes w_n$$

Symmetry $V =$	$w_0 =$	$w_1 =$	$w_2 =$	$w_3 =$	*Symmetry Type*
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$T_3 = (\sigma_3 \otimes \mathbb{1})\,C$	Re w_0	i Im w_1	Re w_2	Re w_3	+TR

Admissibility Conditions

$\Longrightarrow$ exclude *anti*commuting symmetries

Relevance to Classification

$\Longrightarrow$ exclude **unitary, commuting** symmetries

Topological Classification of EM Media

De Nittis & L. (2017)

Non-gyrotropic

$$W = \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} = \begin{pmatrix} \overline{\varepsilon} & 0 \\ 0 & \overline{\mu} \end{pmatrix}$$

$$T_3 = (\sigma_3 \otimes \mathbb{1})\, C$$

Gyrotropic

$$W = \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} \neq \begin{pmatrix} \overline{\varepsilon} & 0 \\ 0 & \overline{\mu} \end{pmatrix}$$

No symmetries

Non-gyrotropic, dual-symm.

$$W = \begin{pmatrix} \varepsilon & -\mathrm{i}\chi \\ +\mathrm{i}\chi & \varepsilon \end{pmatrix} = \begin{pmatrix} \overline{\varepsilon} & -\mathrm{i}\overline{\chi} \\ +\mathrm{i}\overline{\chi} & \overline{\varepsilon} \end{pmatrix}$$

$$T_1 = (\sigma_1 \otimes \mathbb{1})\, C, \; T_3 = (\sigma_3 \otimes \mathbb{1})\, C$$

"**EH**-symmetric"

$$W = \begin{pmatrix} \varepsilon & \chi \\ \chi & \varepsilon \end{pmatrix} = \begin{pmatrix} \overline{\varepsilon} & \overline{\chi} \\ \overline{\chi} & \overline{\varepsilon} \end{pmatrix}$$

$$T_1 = (\sigma_1 \otimes \mathbb{1})\, C$$

Topological Classification of EM Media

De Nittis & L. (2017)

Non-gyrotropic

Class AI

Realized, e. g. dielectrics

Gyrotropic

Class A (Quantum Hall Class)

Realized, e. g. YIG for microwaves

Non-gyrotropic, dual-symm.

Two +TR $\Longrightarrow$ $2 \times$ Class AI

Realized, e. g. vacuum and YIG

[Khanikaev et al, Nature **12** (2013)]

"EH-symmetric"

Class AI

Realized?

(No example known to me.)

4 different topological classes of EM media

Topological Classification of EM Media

De Nittis & L. (2017)

Non-gyrotropic	Gyrotropic
Class AI	Class A **(Quantum Hall Class)**
Realized, e. g. dielectrics	Realized, e. g. YIG for microwaves

Non-gyrotropic, dual-symm.	"**EH**-symmetric"
Two +TR $\Longrightarrow$ $2 \times$ Class AI	Class AI
Realized, e. g. vacuum and YIG	Realized?
[Khanikaev et al, Nature **12** (2013)]	(No example known to me.)

Only one is topologically non-trivial in $d \leq 3$

Consequences of the Classification Result

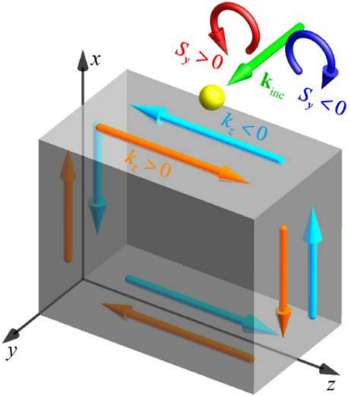
1. Is the Quantum Hall Effect for Light really analogous to the Quantum Hall Effect?
   Answer: **Yes!** Both systems are in **Class A!**
2. Are there other topological effects?
   Answer: **In $d \leq 3$ (unfortunately) no!**
   (E. g. no analog of the Quantum Spin Hall Effect (class AII))

Comparison with Other Works

Bliokh et al's "Quantum Spin Hall Effect for Light"

- Excitation of surface plasmons at 2d metal-dielectric interfaces with circularly polarized waves
- Surface waves have a polarization-dependent transverse component to k vector
- Due to conservation of total angular momentum and fixed polarization of surface plasmon
- *Not* due to presence of **odd time-reversal symmetry** (class AII)!

$\Longrightarrow$ **Not analogous to Quantum Spin Hall Effect in topological sense**

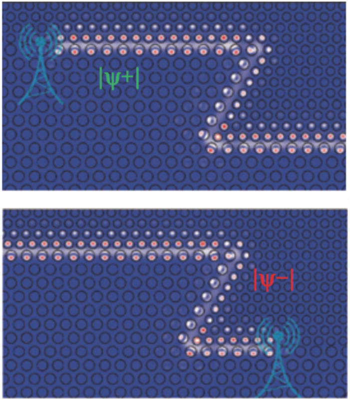


Bliokh et al (2015)

Comparison with Other Works

Unidirectional Modes of Fixed (Pseudo)spin

- Works of Xiao Hu et al and Aleksander Khanikaev et al
- Pseudospin degree of freedom in a **time-reversal-symmetric** medium
- "Hamiltonian" aka Maxwell operator $M = \begin{pmatrix} M_\uparrow & 0 \\ 0 & M_\downarrow \end{pmatrix}$ has a block decomposition
- Topological classification *must be applied to* $M_{\uparrow/\downarrow}$
- $M_{\uparrow/\downarrow}$ of (pseudo) spin ↑/↓ **may not possess** time-reversal symmetry
- Chern numbers $C_\uparrow = -C_\downarrow \neq 0$ possible
- *Not* in contradiction, edge modes come in ↑ / ↓ pairs



Khanikaev et al (2012)

Comparison with Other Works

Inclusion of Crystallographic Symmetries

- Not yet performed
- Schrödinger formalism of Maxwell's equations: ideally suited to transfer ideas from solid state physics to photonics
- But framework used immediately allow transfer of existing quantum techniques to electromagnetism

Thank you for your attention!