Unraveling the Relationship Between Topology and Physics

Max Lein
Advanced Institute of Materials Research, Tohoku University
2016.10.20@Pontificia Universidad Católica de Chile

Goal of Today's Talk

Physical Effect $\longleftrightarrow$ Abstract Mathematics

- Elucidate what "topological" means in the context of physics
- When is topology useful?
- Meaning beyond a buzzword
- Relation of topology to symmetries in physics

The Quantum Hall Effect: the Prototypical System

physical observable $\longleftrightarrow$ *abstract mathematics*

Quantum Hall Effect

$$\sigma^{xy}_{\text{bulk}}(t) \approx \frac{e^2}{h}\,\text{Ch}_{\text{bulk}} = \frac{e^2}{h}\,\text{Ch}_{\text{edge}} \approx \sigma^{xy}_{\text{edge}}(t)$$

transverse conductivity = Chern #

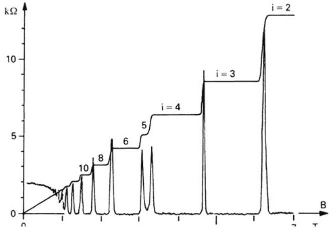
$$\text{Ch}_{\text{bulk/edge}} = \frac{1}{2\pi}\int_{\mathcal{B}} \mathrm{d}k\,\Omega_{\text{bulk/edge}}(k) \in \mathbb{Z}$$

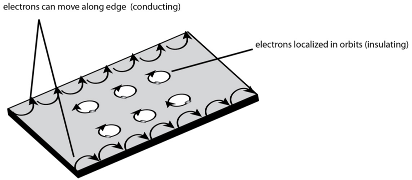
- Edge modes in spectral gaps
- Signed # edge channels $= \text{Ch}(P_{\text{Fermi}})$
- Edge modes unidirectional
- Robust against disorder

***Two* Nobel Prizes**

1980 for experiment: von Klitzing

2016 for theory: Thouless

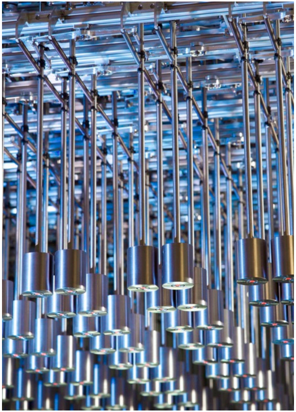




von Klitzing et al (1980)

Topological Insulators for Other Waves: Experiments

Mechanical



Süsstrunk & Huber (2015)

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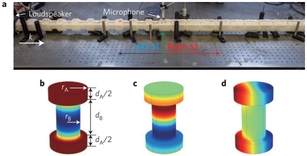

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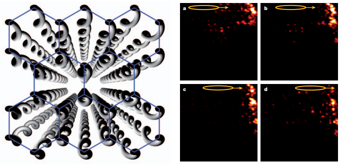
Periodic Waveguide Arrays



Rechtsman, Szameit et al (2013)

Other Effects and Systems Which Are Called Topological

Topological ...

- insulators
- edge or boundary modes
- superconductors
- photonic crystals
- domain walls
- phase transitions
- skyrmions
- …

Characteristic Features of Topological Effects

- **Robustness against perturbations**
- Related to existence or breaking of certain **symmetries**
- Relation of physical effect to "**topological invariants**/numbers"
- Invariants cannot change when **system is deformed continuously** unless either gap closes or localization-delocalization transition happens

When Is a Physical Effect Topological?

Quantum Hall Effect vs. Quantum *Spin* Hall Effect for **Light**

$$\begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla \times \mathbf{H} \\ +\nabla \times \mathbf{E} \end{pmatrix}$$

(dynamical equation)

$$\begin{pmatrix} \nabla \cdot \varepsilon \mathbf{E} \\ \nabla \cdot \mu \mathbf{H} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

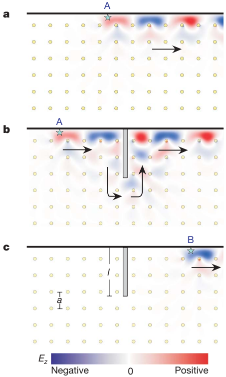
(constraint equation)

Which of the two effects is topological?

In both cases, the authors got it right, though.

When Is a Physical Effect Topological?

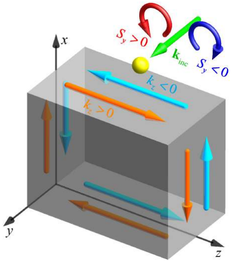
"Quantum Hall Effect"



Joannopoulos, Soljačić et al (2009)

Predicted theoretically by Raghu & **Haldane** (2005)

"Quantum *Spin* Hall Effect"



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When Is a Physical Effect Topological?

In mathematical terms

What is the mathematical structure whose topology is linked to physical effects?

Answer & Explanation

At the end of the talk

How to Axiomatize the Essentials?

Setting here: Quantum Mechanics

1. Hamiltonian operator $H = H^*$ so that the Fermi energy $E_{\mathsf{F}} \notin \mathrm{spec}(H)$ lies in a **spectral gap**
2. Hilbert space $\mathfrak{H}$
3. Schrödinger equation (dynamics)

$$\mathrm{i}\partial_t \psi(t) = H\psi(t), \qquad \psi(0) = \phi \in \mathfrak{H}$$

4. State $P = 1_{(-\infty, E_{\mathsf{F}}]}(H)$
   (all energies up the Fermi energy are occupied)

How to Axiomatize the Essentials?

Question: Existence of a Homotopy

Given two quantum Hamiltonians H_0 and H_1, does there exist a **continuous** path $[0,1] \ni s \mapsto H(s)$ (a *homotopy*) so that

1. $H(0) = H_0$ and $H(1) = H_1$,
2. $H(s)$ has all the same symmetries as H_0 and H_1 (if any), and
3. for all $s \in [0,1]$ the Fermi energy E_{F} lies in a spectral gap

Relevance

If H_0 and H_1 are connected by a homotopy $H(s)$, then *for any topological invariant* T we expect

$$T(H_0) = T(H(s)) = T(H_1).$$

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Topology & Continuity

Definition (Topological Space)

Let X be any set and $\mathcal{T} = \{U_j \subseteq X\}_{j \in \mathcal{I}}$ a collection of subsets. Then $(X, \mathcal{T})$ is a **topological space** if and only if:

1. $\emptyset, X \in \mathcal{T}$
2. For *any* (finite or infinite) subcollection $\mathcal{J} \subseteq \mathcal{I}$ the family satisfies $\bigcup_{j \in \mathcal{J}} U_j \in \mathcal{T}$.
3. For any *finite* collection $\mathcal{J} \subseteq \mathcal{I}$ the *intersection* satisfies $\bigcap_{j \in \mathcal{J}} U_j \in \mathcal{T}$.

The U_j are called **open sets**, and $\mathcal{T}$ is said to give a **topology** to X.

Topology & Continuity

Definition (Continuous function)

Let X and Y be topological spaces. A map $f : X \longrightarrow Y$ is continuous if the *preimage* of an open set in Y is an open set in X.

Definition (Homeomorphism)

A map $f : X \longrightarrow Y$ between topological spaces is a homeomorphism if and only if

1. f is continuous and
2. its inverse f^{-1} exists and is continuous as well.

In that case X and Y are called homeomorphic.

If X and Y are homeomorphic, then Y can be thought of as being obtained from X by deformation without cutting, tearing or gluing.

Topology & Continuity

Three-line summary

- Topology gives rise to the notion of continuity
- Different choices of topology, different types of continuity
- In topology homeomorphic spaces are usually considered to be “one and the same”, e. g. the circle is homeomorphic to a square and a triangle

Homotopy

Definition (Homotopy)

Two maps $f_0, f_1 : X \longrightarrow Y$ are said to be homotopic if there exists a continuous map $g : [0,1] \times X \longrightarrow Y$ such that $g(0) = f_0$ and $g(1) = f_1$. In that case we call f_0 and f_1 homotopic.

Definition (Homotopy equivalence)

Two topological spaces X and Y are called homotopically equivalent if there exist continuous maps $f : X \longrightarrow Y$ and $g : Y \longrightarrow X$ so that $f \circ g$ is homotopic to id_X and $g \circ f$ is homotopic to id_Y.

Homotopy

- Homotopy equivalence of spaces is *weaker* than homeomorphy, i. e. there are examples where X and Y are homotopically equivalent, but not homeomorphic.
- If X and Y are homotopically equivalent, then one can think of obtaining Y from X by deforming it without gluing, cutting and tearing, and in addition to blowing it up or contracting it.

Quantifying Topology

How to decide whether two spaces are equivalent?
Very difficult problem, depends on all of the details.

Quantifying Topology

How to decide whether two spaces are not equivalent?

Much easier question.

$\rightsquigarrow$ Use groups associated to topological spaces.

The Fundamental Group

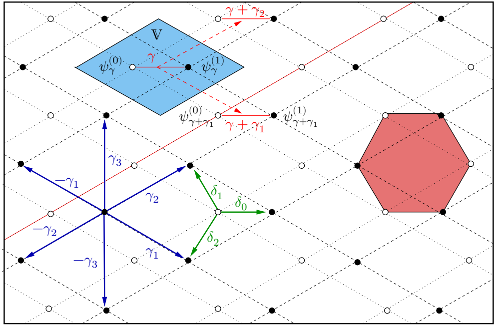
Definition (The Fundamental group)

Let X be a topological space, and $x_0 \in X$ a point. Then the fundamental group $\pi_1(X, x_0)$ consists of *homotopy classes* of maps $f : [0, 1] \longrightarrow X$ with $f(0) = x_0 = f(1)$.

An Example from Physics: A Simple Model for Graphene

$$H(q_1, q_2, q_3) = \begin{pmatrix} q_3 & 1 + q_1\,\mathfrak{s}_1 + q_2\,\mathfrak{s}_2 \\ 1 + q_1\,\mathfrak{s}_1^* + q_2\,\mathfrak{s}_2^* & -q_3 \end{pmatrix}$$

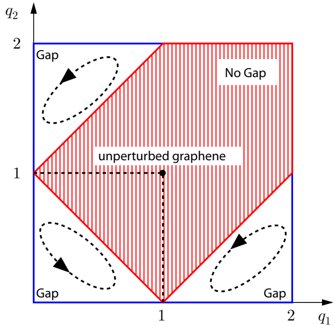
q_1, q_2 hopping parameters, q_3 stagger potential



An Example from Physics: A Simple Model for Graphene

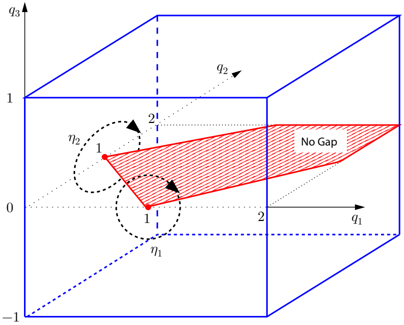
Periodic Deformations

No stagger potential ($q_3 = 0$)



$$\pi_1(X) = \{0\}$$

With stagger potential ($\propto q_3$)



$$\pi_1(X) = \mathbb{Z}^2$$

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To be continued ...

Computable But Not *Algorithmically* Computable

- Solution to classification problem seems clear: compute homotopy classes for Hamiltonians
- Unfortunately, homotopy classes which make up e. g. $\pi_1(X)$ are **not algorithmically computable**
- Previous example: computation by "eyeballs"
- More difficult in complicated, possibly infinite-dimensional spaces!

Solution: Compute *other* groups associated to topological spaces

Algorithmically Computable Criteria

- Leads to e. g. K-groups and cohomology groups
- Too complicated to explain in detail here
- *Groups* algorithmically computable, and algorithms known and (in some cases) implemented numerically
- Side note: Same mathematics gives very powerful for topological data analysis (rigorous (!) approach to analysis of e. g. granular matter & glasses)
- Does *not* necessarily answer the question: **What homotopy class does a system belong to?**

Idea of Topological Invariant

- T is a map from a class of spaces (e. g. vector bundles) to typically $\mathbb{Z}$, $\mathbb{Z} \bmod p$
- T explicitly computable

Prototypical Theorem

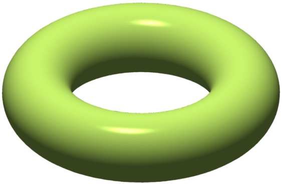
Let X and Y be two topological spaces. If $T(X) \neq T(Y)$, then X and Y are *not homeomorphic*.

The Genus

The Genus

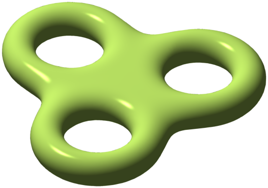
Genus g is the “number of holes”

$g = 1$



$g = 2$

$g = 3$



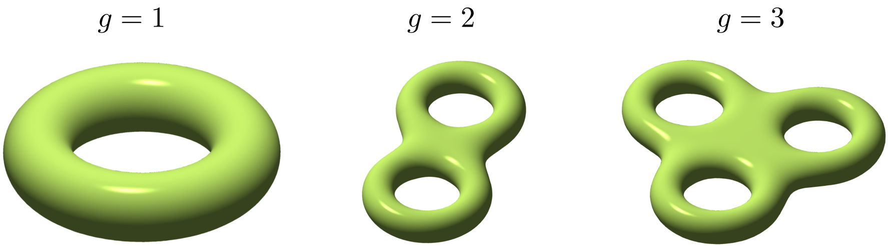
The Genus

Theorem (Gauß-Bonnet)

Let X be a two-dimensional, compact, orientable manifold without boundary, and K its Gaußian curvature. Then we have

$$\int_X \mathrm{d}A\, K = 4\pi(1-g).$$

Topology $\longleftrightarrow$ geometry of $2d$ manifolds, g computable!



Chern Numbers and Vector Bundles

Complex Vector Bundles

Suppose $\mathfrak{H}(x)$ is a family of complex Hilbert spaces of the same dimension that depend continuously on $x \in X$. Then the vector bundle of rank $m = \dim \mathfrak{H}(x)$

$$\mathcal{E} : \bigsqcup_{x \in X} \mathfrak{H}(x) \longrightarrow X$$

is the space obtained by "gluing together" the $\mathfrak{H}(x)$ over X.

Chern Numbers and Vector Bundles

Example (The Bloch Bundle)

Let $P = 1_{(-\infty\,,\,E_{\mathrm{F}}]}(H)$ be the Fermi projection of a periodic Hamiltonian $H = \frac{1}{2m}(-\mathrm{i}\hbar\nabla)^2 + V_{\mathrm{per}}$.

- H and P admit a Bloch-Floquet decomposition (band picture)
- $H(k)$ and $P(k)$ depend analytically on Bloch momentum k (gap condition!)
- Brillouin zone $\mathcal{B} \cong \mathbb{T}^d$
- Bloch bundle $\mathcal{E}_{\mathrm{B}}(P) := \bigsqcup_{k \in \mathbb{T}^d} \mathrm{ran}\, P(k) \longrightarrow \mathbb{T}^d$

Chern Numbers and Vector Bundles

In analogy to the genus: Chern numbers distinguish between inequivalent vector bundles

Theorem (Prototypical Statement)

If $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *are two vector bundles over* $\mathbb{T}^d$ *of the same rank. Then* $\mathrm{Ch}_j(\mathcal{E}_1) \neq \mathrm{Ch}_j(\mathcal{E}_2)$ *implies* $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *are not equivalent.*

Chern Numbers and Vector Bundles

In analogy to the genus: Chern numbers distinguish between inequivalent vector bundles

Theorem (Classifciation of Complex Vector Bundles)

If $\mathcal{E}_1$ *and* $\mathcal{E}_2$ *are two vector bundles over* $\mathbb{T}^d$ *of the same rank, and* $d = 1, 2, 3, 4$. $\mathcal{E}_1$ *are equivalent* $\mathcal{E}_2$ *if and only if all Chern numbers of the two bundles agree.*

Chern Numbers and Vector Bundles

In analogy to the genus: Chern numbers distinguish between inequivalent vector bundles

Formula for (First) Chern Numbers

$$\mathrm{Ch}_{jl}(P) = \frac{1}{2\pi} \int_{\mathbb{T}^2_{jl}} \mathrm{d}k_j \wedge \mathrm{d}k_l \, \mathrm{Tr}\Big(P(k) \left[\partial_{k_j} P(k) \, , \, \partial_{k_l} P(k) \right] \Big)$$

Example Continued: Periodically Deformed Graphene

Periodic deformation $\equiv$ loop $q(t)$ in parameter space

$$H(t) = H(q(t)) = \begin{pmatrix} q_3(t) & 1 + q_1(t)\,\mathfrak{s}_1 + q_2(t)\,\mathfrak{s}_2 \\ 1 + q_1(t)\,\mathfrak{s}_1^* + q_2(t)\,\mathfrak{s}_2^* & -q_3(t) \end{pmatrix}$$

q_1, q_2 hopping parameters, q_3 stagger potential

Homotopy-invariance of Chern numbers

If $q(t)$ and $q'(t)$ (and thus, $H(t)$ and $H'(t)$) are in the same homotopy class, then

$$\mathrm{Ch}_j(P) = \mathrm{Ch}_j(P')$$

where $P = 1_{(-\infty, E_\mathrm{F}]}(H)$ and $P' = 1_{(-\infty, E_\mathrm{F}]}(H')$.

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De Nittis-L. (2011)

1. Compute Chern numbers for fundamental loops η_1 and η_2 which generate $\pi_1(X)$.
2. Determine homotopy class of $q(t) \rightsquigarrow [q(t)] = (n_1, n_2)$.
3. Charge accumulated over one period in spatial direction j

$$\Delta C = n_1\,\mathrm{Ch}_{tj}(\eta_1) + n_2\,\mathrm{Ch}_{tj}(\eta_2)$$

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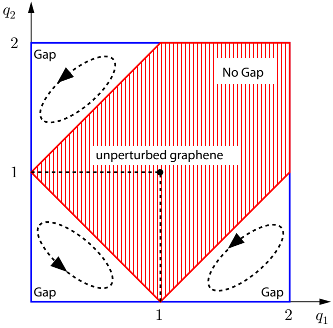

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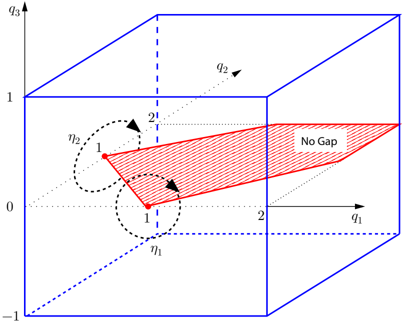

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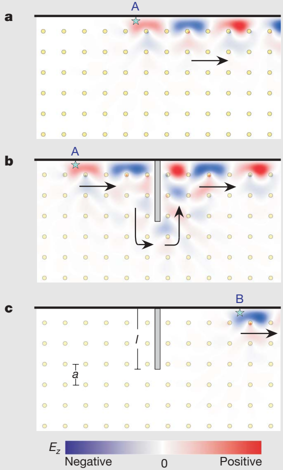
The Quantum Hall Effect for Light

Predicted theoretically by Raghu & **Haldane** (2005) ...

$$\left.\begin{pmatrix} \overline{\varepsilon} & 0 \\ 0 & \overline{\mu} \end{pmatrix} \neq \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix}\right\} \Longrightarrow$$

symmetry breaking



Joannopoulos, Soljačić et al (2009)

Schrödinger Formalism of Electromagnetism

$$\left.\begin{matrix} \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla \times \mathbf{H} \\ +\nabla \times \mathbf{E} \end{pmatrix} \\ \text{dynamical Maxwell equations} \end{matrix}\right\} \Longleftrightarrow \left\{\begin{matrix} \mathrm{i}\partial_t \Psi = M \Psi \\ \text{“Schrödinger-type equation”} \end{matrix}\right.$$

$$\Psi(t) = \big(\mathbf{E}(t), \mathbf{H}(t)\big) \in \mathfrak{H} = \Big\{ \Psi \in L^2_W(\mathbb{R}^3, \mathbb{C}^6) \; \big| \; \Psi \text{ transversal} \Big\}$$

$$M = \underbrace{\begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix}^{-1}}_{=W^{-1}} \underbrace{\begin{pmatrix} 0 & +(-\mathrm{i}\nabla)^{\times} \\ -(-\mathrm{i}\nabla)^{\times} & 0 \end{pmatrix}}_{=\mathrm{Rot}} = M^*$$

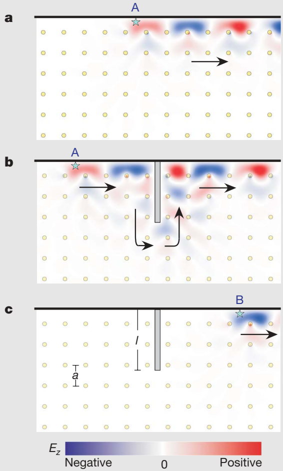
$$\left.\begin{matrix} \text{Maxwell equations} \\ \Longleftrightarrow \\ \text{Maxwell operator } M = M^* \end{matrix}\right\} \Longrightarrow$$

Adaptation of **techniques from quantum mechanics** to electromagnetism

Topological Insulators for Light

Predicted theoretically by Raghu & **Haldane** (2005) ...

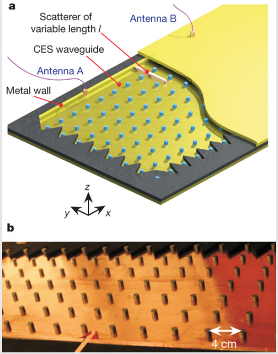
$$\left.\begin{pmatrix} \bar{\varepsilon} & 0 \\ 0 & \bar{\mu} \end{pmatrix} \neq \begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} \quad \text{symmetry breaking} \right\} \Longrightarrow$$



Joannopoulos, Soljačić et al (2009)

Topological Insulators for Light

... and realized experimentally by Joannopoulos et al (2009)



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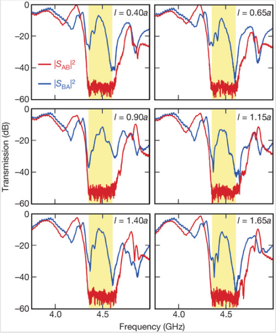

Joannopoulos, Soljačić et al (2009)

Topological Insulators for Other Waves: Experiments

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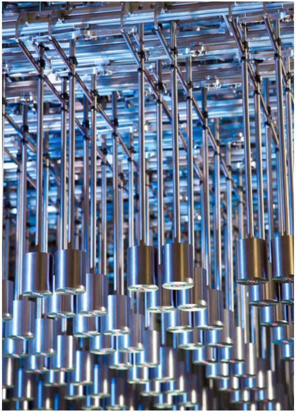

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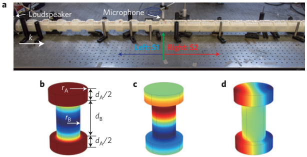

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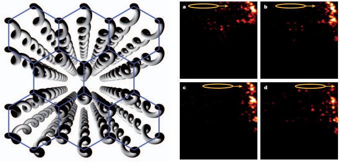
Periodic Waveguide Arrays



Rechtsman, Szameit et al (2013)

Shared Mathematical Structure of these Wave Equations

Classical electromagnetism

$$\begin{pmatrix} \varepsilon & 0 \\ 0 & \mu \end{pmatrix} \frac{\partial}{\partial t} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} -\nabla \times \mathbf{H} \\ +\nabla \times \mathbf{E} \end{pmatrix} - \begin{pmatrix} j \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \nabla \cdot \varepsilon \mathbf{E} \\ \nabla \cdot \mu \mathbf{H} \end{pmatrix} = \begin{pmatrix} \rho \\ 0 \end{pmatrix}$$

Spin waves

$$\mathrm{i} \frac{\partial}{\partial t} \begin{pmatrix} \beta(k) \\ \beta^\dagger(-k) \end{pmatrix} = \sigma_3 \, H(k) \begin{pmatrix} \beta(k) \\ \beta^\dagger(-k) \end{pmatrix}$$

Transverse acoustic waves

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \mathbf{v} \end{pmatrix} = \begin{pmatrix} 0 & -\nabla \rho_0 \\ -\rho_0^{-1} \nabla \gamma v_s^2 & 0 \end{pmatrix} \begin{pmatrix} \rho \\ \mathbf{v} \end{pmatrix}$$

Characteristics

- Linear (to leading order)
- First order in *time*
- Can be rewritten in form of **Schrödinger equation**

Other examples

Plasmons, magnetoplasmons, van Alvén waves, etc.

Explanation via **Bulk-Boundary Correspondences**

$$O_{\text{bulk}}(t) \approx T_{\text{bulk}} = T_{\text{edge}} \approx O_{\text{edge}}(t)$$

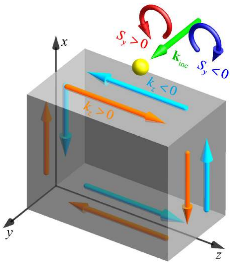
concrete physics $\longleftrightarrow$ **abstract mathematics**

1. Provide a **first-principles derivation of effective dynamics** in photonic crystals and periodic waveguide arrays.
   De Nittis & L., Commun. Math. Phys. **332**, 221–260, 2014
2. Understand the roles symmetries and various waveguide geometries play.
   De Nittis & L., Annals of Physics **350**, 568–587, 2014
3. Find **bulk-edge correspondences** in periodic light conductors, i. e. relations between dynamical and topological quantities.
   In progress

The Quantum Spin Hall Effect for Light

Locking of Surface Mode's Transverse Momentum to Spin

- Due to conservation of total angular momentum
- Surface modes necessarily linearly polarized
- Conversion of spin to *orbital* angular momentum of the surface wave
- No topological origin
- Authors made the correct claim



Bliokh, Smirnova & Nori (2015)

Summary

- The relationship between topology and physics has to be made concrete on a **case-by-case basis**.
- Identification of the mathematical object whose topology is relevant (e. g. vector bundles)
- The methods with which the topology is analyzed lead to **algorithmically computable criteria**.

Max's Criteria

When Do I Call a Physical Effect "Topological"?

- When the object whose topology is relevant has been identified.
- When topology helps to understand the effect's mechanism.

In case of doubt: ask a mathematical physicists you trust. (We are happy whenever we find applications to real physical effects!)

Thank you for your attention!